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How non-uniform tolerance parameter strategy changes the response of scale-free networks to failures

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Abstract. In this paper, we introduce a non-uniform tolerance parameter (TP) strategy (the tolerance parameter is characterized by the proportion between the unused capacity and the capacity of a vertex) and study how the non-uniform TP strategy influences the response of scale-free (SF) networks to cascading failures. Different from constant TP in previous work of Motter and Lai (ML), the TP in the proposed strategy scales as a power-law function of vertex degree with an exponent b. The simulations show that under low construction costs D, when $b > 0$ the tolerance of SF networks can be greatly improved, especially at moderate values of b; When $b < 0$ the tolerance gets worse, compared with the case of constant TP in the ML model. While for high D the tolerance declines slightly with the b, namely $b < 0$ is helpful to the tolerance, and $b > 0$ is harmful. Because for smaller b the cascade of the network is mainly induced by failures of most high-degree vertices; while for larger b, the cascade attributes to damage of most low-degree vertices. Furthermore, we find that the non-uniform TP strategy can cause changes of the structure and the load-degree correlation in the network after the cascade. These results might give insights for the design of both network capacity to improve network robustness under limitation of small cost, and for the design of strategies to defend cascading failures of networks.

PACS. 89.75.Hc Networks and genealogical trees – 89.75.Fb Structures and organization in complex systems – 89.40.Bb Land transportation – 89.20.Hh World Wide Web, Internet

1 Introduction

Complex networks have attracted a great deal of attention as an important tool to describe various complex systems in nature and society. It has been disclosed by recent studies that many networks are characterized by scale-free networks where the fraction of vertices having k connections follows a power-law distribution, $P(k) \sim k^{-\gamma}$ with $2 \leqslant \gamma \leqslant 3$, such as the Internet and WWW networks [1,2].

The security of complex networks is one of the key problems to guarantee networks in function. Cascading failures, triggered by a small initial shock or attacks, are common in most of real complex networks, such as power grid (blackout), Internet (packet congestion or server down), traffic and transportation system (vehicles jams), and so forth [1–16]. A number of important aspects of cascading failures in complex networks have been widely discussed in both artificially generated scale-free (SF) topologies and real world networks with scale-free properties by static and dynamic approaches [3–17]. These works pointed out that scale-free networks are fragile to intentional attacks, since some nodes are much more important than others. It is important to understand how to design networks preventing from failures and attacks.

Recently, a typical dynamical model developed by Motter and Lai (ML) [8], incorporates dynamics of the flow of physical quantities on the network. In the model, each vertex is assigned a finite capacity, given as $C_i = (1 + \alpha_i)l_i$, where the tolerance parameter (TP) $\alpha_i = \alpha$ is constant, the initial load l_i of vertex i is defined as the sum of the number of data packets passing through the vertex when a unit data packet is transmitted between each pair of vertices. The capacity C_i of vertex i is the maximal load that the vertex can handle, and vertex i will fail when its real load exceeds the capacity C_i. As known, heterogeneity is an essential character of SF networks, which is highly heterogeneous in connectivity patterns and load distributions, decaying as power-law regimes. And even in collapsed networks, degree distributions and load

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distributions are highly heterogeneous and follow power laws [13]. Obviously, heterogeneously distributed load or capacity is deeply related to not only the intrinsic dynamics of packet flow, but also the size of cascade as the consequence of the propagation of overload failures. Since loads in the collapsed SF networks still distribute uniformly as mentioned above, the tolerance might be related to the distribution of TP. Instead of homogeneous TP in the ML model, we expect that non-uniform TP strategies would make the network more robust under limitation of a given budget D (the given budget D is defined as the sum of all vertices' capacity). More recently, based on the ML model, few studies about tolerance control and defense have been reported. Motter [18] developed a costless strategy of defense based on a selective further removal of nodes and edges in SF networks. Hayashi et al. [19,20] introduced a defense strategy based on emergent rewirings between neighbors of the attacked node, and investigated the size of cascade on SF networks with controllable correlations. However, no publication is focused on the impact of the non-uniform TP on the robustness of SF networks. In the present study, we introduce a non-uniform tolerance parameter (TP) strategy in cascading failures, where the TP α_i is not uniform for all vertices but depends on degree k_i, and aim to analyze how the proposed strategy influences the dynamical cascading process on scale-free networks, in terms of the tolerance and the critical values. This may offer some implications to reinforce and design networks.

2 The non-uniform tolerance parameter strategy

Initially, assume one unit of physical quantities (data packets or flows) is transmitted between each pair of vertices along the shortest path. If there is more than one shortest path, then data packets are evenly balanced on them. When the network is attacked intentionally, the highest loaded or degree vertex is destroyed or fails, the shortest paths between each pair of remaining vertices are changed, and then loads on these vertices are rearranged. If the loads of some vertices surpass their capacities, these vertices will overload and fail at the same time. After that, the distribution of the shortest paths is changed, and subsequent failures occur until no vertex fails again or all remaining vertices are disconnected. In the previous ML model, the tolerance parameter (TP) is defined as the proportion that the capacity is beyond the initial load of a vertex and a constant TP is used. As the given capacities of vertices are limited by construction cost D, the distribution of the capacities is strongly correlated with the size of collapsed network. In the other words, the distribution of TP would have vital impacts on the propagation of failures. Since the rearranged loads of vertices are still proportional to the power of their degrees, a general TP strategy is considered, where the TP of a vertex i is dependent on the degree, given as

$$\alpha_i = a\,(k_i/k_{\max})^b \tag{1}$$

where a and b is parameters, and $k_{\max}$ is the maximal degree on the network. For $b > 0$ ($b < 0$), vertices with higher (smaller) degrees have larger TP. When $b = 0$, the case represents the uniform TP, equal to that of the ML model. Here the capacity C_i of vertex i, which is related to both its initial degree k_i and its initial load l_i, given as

$$C_i = (1+\alpha_i)l_i = \left[1 + a\,(k_i/k_{\max})^b\right] l_i. \tag{2}$$

3 Simulations and results

In our studies, we construct the scale-free networks according to the Barabasi-Albert (BA) model [21]. Assume the vertex with the heaviest load is removed from the network and cascade failures happen. Let us investigate the difference between the tolerance of SF networks under uniform TP α and non-uniform TP α_i given by equation (1). To compare with the uniform TP case, equivalent uniform TP α is calculated as follows to guarantee the same allocated budget D in the network under the two cases,

$$\alpha = \frac{\sum_{i=1}^{N} \alpha_i l_i}{\sum_{i=1}^{N} l_i} = \frac{\sum_{i=1}^{N} a(k_i/k_{\max})^b l_i}{\sum_{i=1}^{N} l_i}. \tag{3}$$

Cascading failures can be measured conveniently by the following ratio

$$G = \frac{N'}{N} \tag{4}$$

where N and N' are the numbers of vertices in the largest connected component before and after the cascade, respectively. The network maintains its integrity if $G \approx 1$, while breakdown at a global scale occurs if $G \approx 0$ [8,13].

In Figure 1a, we present the ratio G as a function of b at the equivalent $\alpha = 0.05$, 0.1, 0.2, 0.3, 0.4 (which is calculated as Eq. (3)) on SF networks with $\gamma = 3$. To make clear the distinct between the two cases, a relative error Err_G of the G is defined as follows,

$$Err_G = \frac{G(b) - G(0)}{G(0)} \tag{5}$$

where $G(b)$ denotes the ratio G at b. The relative error represents the relative amount that the size of the largest connected component is improved, comparing with the uniform case. A positive Err_G means that there is an improvement of the tolerance, and a negative value means that a stronger damage occurs. The relative error for different b is displayed in Figure 1b. From Figures 1a, 1b, we observe that the ratio G is a convex-shape function of b, e.g. for $\alpha = 0.05$, G grows from 0.02 at $b = 0$ to a local maximal value about 0.2 at $b = 0.5$ (ten times increment), and then starts to drop with b. It seems that in small α (low budget D) adjusting b can observably enhance the tolerance of SF networks. Whereas for large α, the amplitude of Err_G descends with the increase of b.

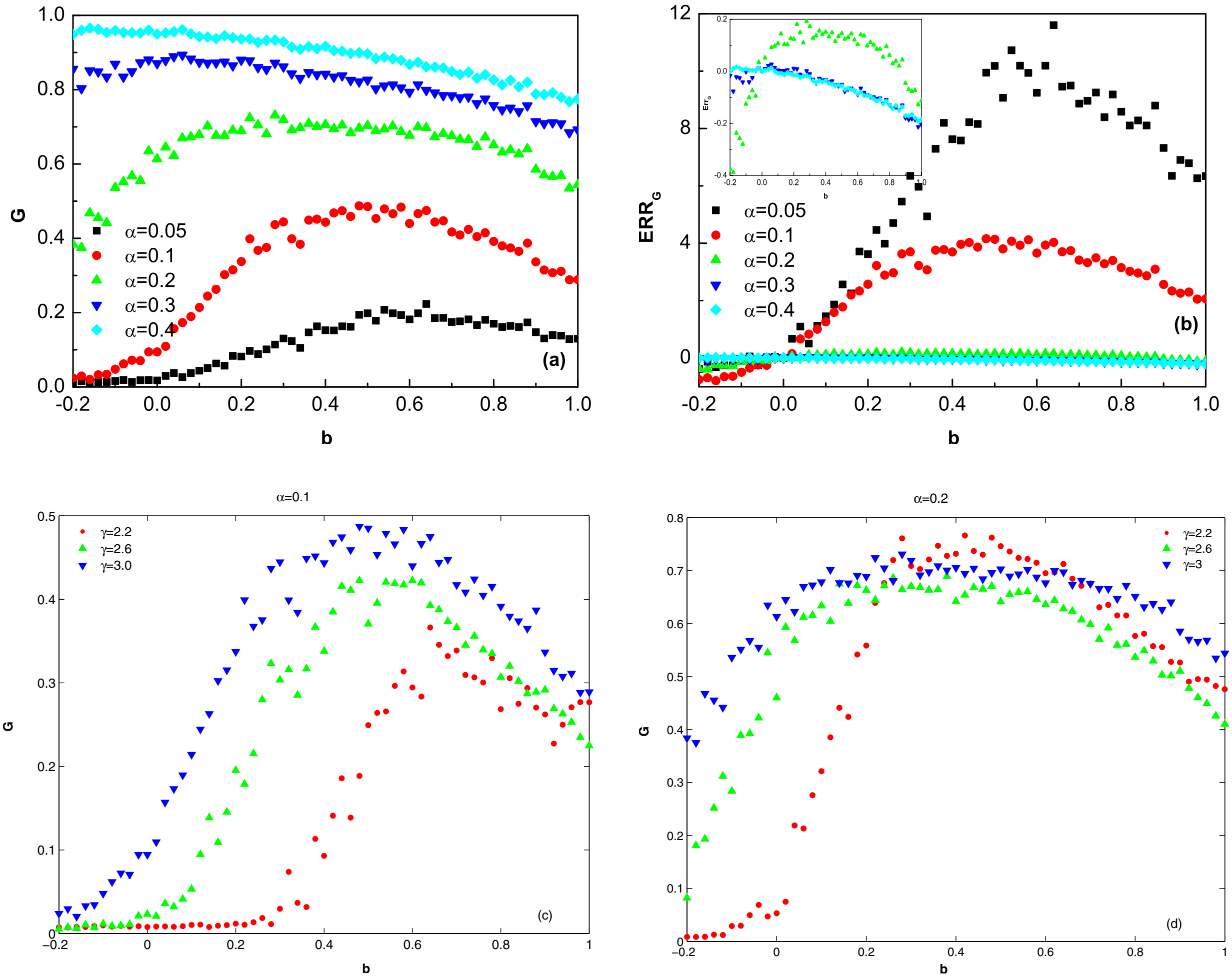

Fig. 1. (a) The ratio G versus the parameter b at the equivalent uniform TP $\alpha = 0.05$, 0.1, 0.2, 0.3, 0.4 on SF networks with the exponent $\gamma = 3.0$. (b) The relative error of the ratio G versus the b. The ratio G versus the parameter b at the equivalent uniform TP $\alpha = 0.1$ (c) and $\alpha = 0.2$ (d) on SF networks with different exponents $\gamma = 2.2$, 2.6, 3.0. In SF networks, the number of vertices is 500, and the average links of each vertex is 4. The simulations are averaged for 50 realizations.

This indicates that benefit from the non-uniform TP strategy will graduate away with α. The relationship between G and b in SF networks with different exponents $\gamma = 2.2$, $\gamma = 2.6$ (which are generated by the extended BA-model in Ref. [22]) and $\gamma = 3$ are shown in Figures 1c, 1d. From Figures 1c, 1d, it seems that the effects of the non-uniform TP strategies are very similar for different exponents in the range $\gamma \in (2, 3]$. Thus, the non-uniform TP strategy has an important effect on the tolerance of SF networks, and parameter b plays a key role in the effect.

To clarify the effect of the non-uniform TP strategy, Figures 2a–2c and 3a–3c show degree distribution $P'(k)$ after the failures and its normalized distribution $P''(k) = P'(k)k^{-b}$ for different b, respectively. The degree distribution approximates as a power-law regime $P'(k) \sim k^{-\gamma 1}$ with a scaling exponent $\gamma 1$. The exponent $\gamma 1$ has a decreasing tendency with b, and this is directly observed in the overlapping curves of the normalized distribution. These results suggest that for lower b, cascading failures of the network can mainly ascribe to the breakdown of most high-degree vertices. While for larger b, failures attribute to the damage of most low-degree vertices. To make these clear, Figures 4, 5 display variations of total number ratio, number ratio at each time-step and average degree of failed nodes with time step for $\alpha = 0.2$ and $\alpha = 0.4$. Total number ratio of failed nodes is defined as the ratio between the number of total failed nodes and the number of total nodes of the network. Number ratio of failed nodes at each time-step is defined as the ratio between the number of failed nodes at each time-step and the number of total nodes of network. From Figure 4, it can be seen that during the cascading process, with the increase of b the number of failed nodes at each time step initially increases, and then decreases gradually at the following time-step. But during the whole process, average degree of failed nodes always decreases with the increase of b.

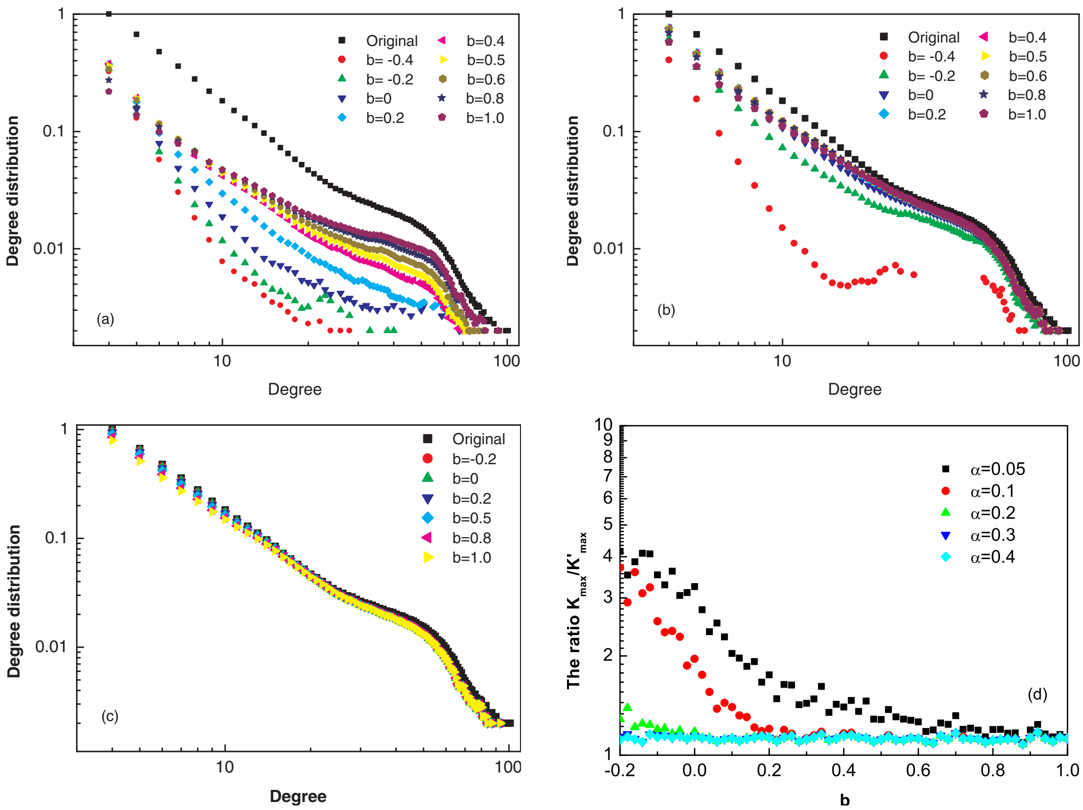

Fig. 2. The degree distribution for different α in a double logarithmic axis (a) $\alpha = 0.05$, (b) $\alpha = 0.2$, (c) $\alpha = 0.4$, (d) the ratio $k_{\max}/k'_{\max}$ of the maximal degree before and after the removal versus the b.

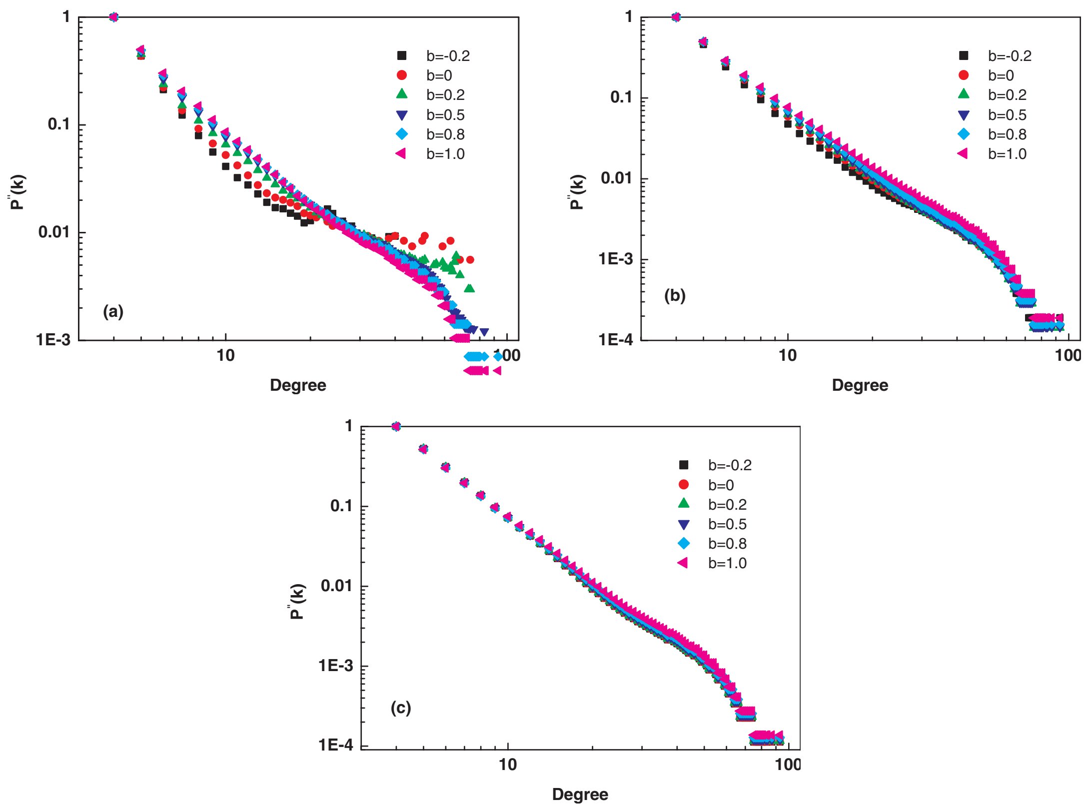

Fig. 3. The normalized degree distribution $P''(k)$ for different α (a) $\alpha = 0.05$, (b) $\alpha = 0.2$, (c) $\alpha = 0.4$.

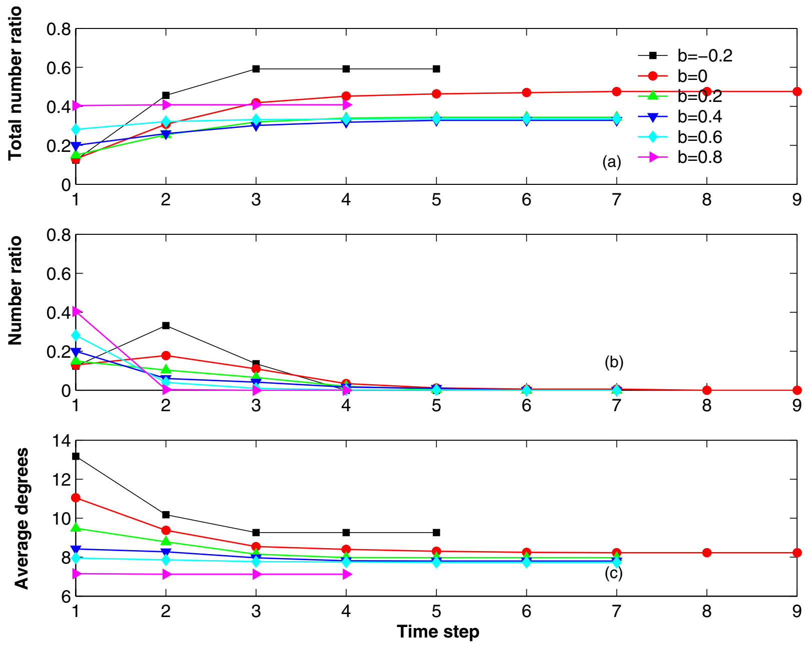

Fig. 4. The total number ratio (a), number ratio at each step (b) and average degree (c) of failures nodes vary with time step for $\alpha = 0.2$ on SF networks with the exponent $\gamma = 3.0$.

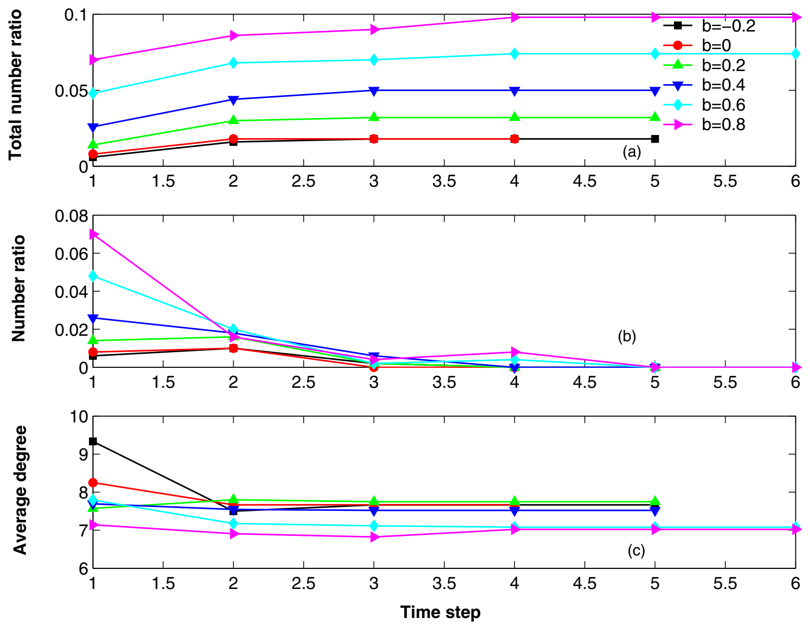

Fig. 5. The total number ratio (a), number ratio at each step (b) and average degree (c) of failures nodes vary with time step for $\alpha = 0.4$ on SF networks with the exponent $\gamma = 3.0$.

This indicates that at start stage the speed of failures for lower b is faster than that for larger b. Subsequently, the opposite appears. However, in the whole more high-degree vertices fail for lower b, while for larger b more low-degree vertices fail. For lower b, highly connected vertices have no enough capacities carrying the redistributed load at first. And the increment of b will lead to the increases of capacities in high-degree vertices, which make some of high-degree vertices escaping from the failures. Thus the number of high-degree vertices survived grows with b. Since generally the load is proportional to the degree, the amount of rearranged loads induced by failures of high-degree vertices is dramatically reduced, and then other high-degree vertices failing decrease further. The opposite proceeds for low-degree vertices. These differences between high-degree and low-degree vertices are justified by the conclusions drawn from Figures 6a–6c and 7a–7c, where the correlation $L(k), L'(k)$ between load and degree before failures, after failures and its normalized correlation $L''(k) = L'(k)k^{\frac{b}{20\alpha}}$ for different b are shown.

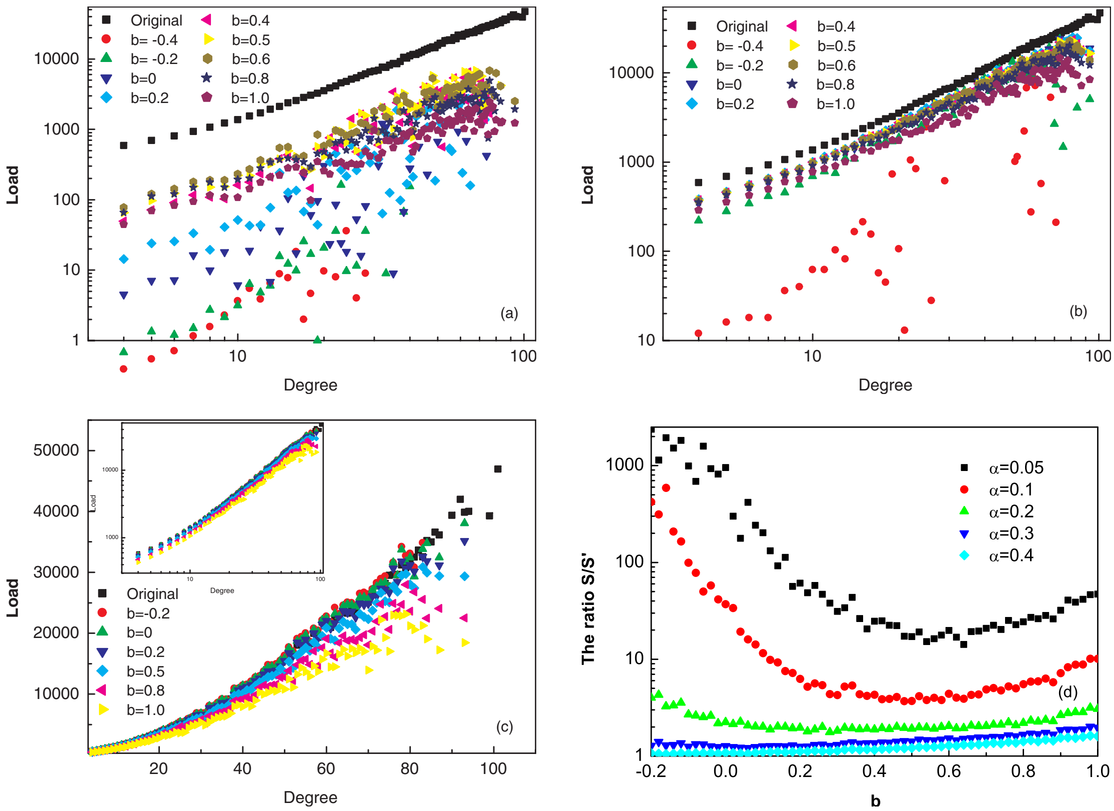

Fig. 6. The correlation between the load and the degree for different α. (a) $\alpha = 0.05$, (b) $\alpha = 0.2$, (c) $\alpha = 0.4$, (d) the ratio S/S' of the total load before and after the removal versus the b.

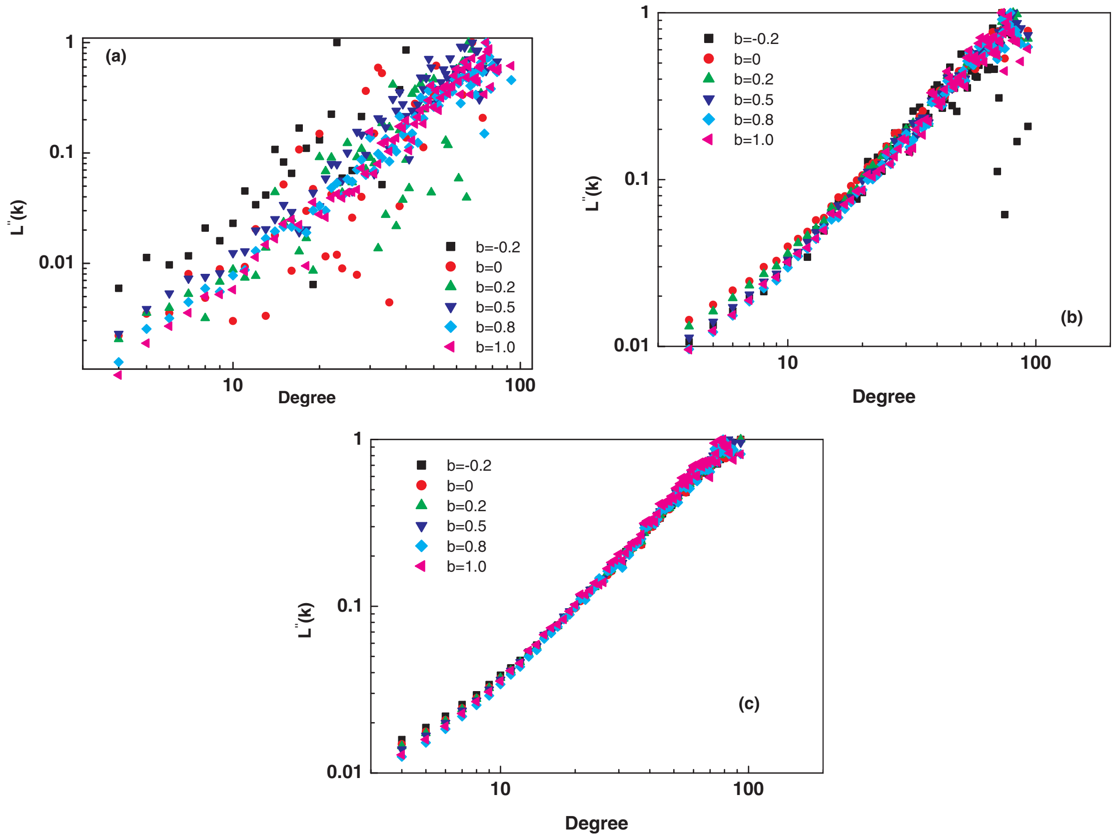

Fig. 7. The normalized correlation $L''(k)$ for different α (a) $\alpha = 0.05$, (b) $\alpha = 0.2$, (c) $\alpha = 0.4$.

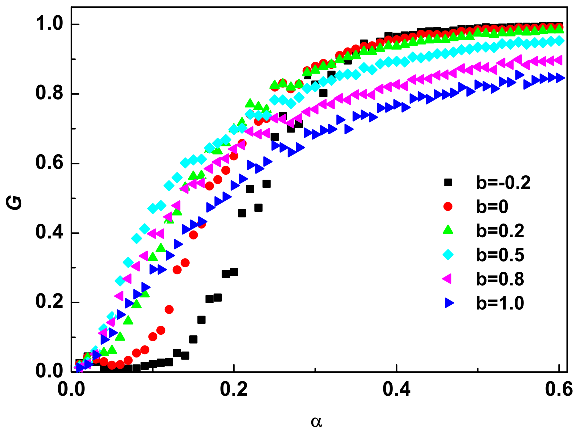

Fig. 8. The ratio G versus the equivalent uniform TP α on SF networks with the exponent $\gamma = 3.0$.

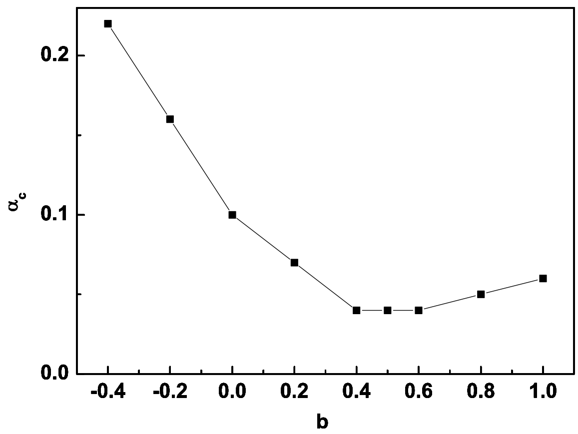

Fig. 9. The critical value α_c versus the parameter b on SF networks with $\gamma = 3.0$.

It is observed that similar to the correlation $L(k)$ before failures, the correlation after failures follows a power-law regime $L'(k) \sim k^{\eta 1}$ with an exponent $\eta 1$. The exponent $\eta 1$ decreases with b. As b grows, high-degree vertices bear less portion of the total load. At the same time, capacities in high-degree vertices go up with b, thus less high-degree vertices are destroyed during the failures, opposite to low-degree vertices. These are in good agreement with our conjecture. These findings just offer sufficient evidences for the relationship between b and the tolerance shown in Figure 1. For low α, the initial increase of b leads to an increment of G, because the number of the low-degree vertices failing is less than that of high-degree vertices survived. After the local peak of G, a further increase of b results in that overloaded low-degree vertices develop beyond high-degree vertices escaping from the failures. Then the ratio G starts to drop with b. This also can be verified by variation of total load in Figure 2d (the ratio of the total load before and after the removal versus b) and the maximal degree before and after the cascade in relation to b for the network in Figure 6d. From Figure 5, it can be obvious that the situation with large α is similar to that with low α. During the cascading process the lower b is, the more high-degree vertices fail, the less low-degree vertices fail. Different from low α, with the increase of b the speed of failures propagation becomes slow. Wholly the growth of capacity for high-degree vertices is much faster than that for low-degree vertices, resulting in more protection of high-degree vertices and more damage of low-degree vertices. Thus, when failures of low-degree vertices surpass surviving high-degree vertices with α, the decrease of b will be helpful to slightly enhance the robustness of low-degree vertices and the tolerance of the network to some extent. The simulations above indicate that non-uniform TP strategy can no only bring improvements on the tolerance of the network, but also change the structure and the load-degree correlation of SF networks after the failures.

The transition occurs in the ML model at a critical value α_c, below which the network will disintegrate completely. Here, we numerically investigate how the non-uniform TP strategy affects the critical value of the equivalent α_c. Figure 8 displays the relationship between G and α for different b on SF networks with $\gamma = 3$. Also we can observe that transition phenomena occur in the non-uniform TP case. The critical points change with b. Figure 9 shows the equivalent critical value α_c versus b. Similar to the results in reference [13], the critical value α_c is about 0.1 for the uniform TP case ($b = 0$). In comparison with the uniform TP case ($b = 0$), the critical value α_c is reduced from $\alpha_c = 0.1$ at $b = 0$ to 0.024 at $b = 0.5$, about four times. The improvement is strongly dependent on b. The critical value α_c decreases with the increase of b at the beginning. With the further increase of b, the critical value α_c changes little. These numerical results coincide with those numerical analysis obtained in Figure 1. These findings show that the non-uniform TP strategy can change the critical point of the transition phenomena, and the critical value α_c can be minimized via adjusting b.

4 Conclusions

In summary, we have studied tolerance of SF networks under the non-uniform tolerance parameter (TP) strategy, where TP scales as vertex degree with an exponent b. The simulation results indicate that the non-uniform TP strategy has an important effect on the tolerance of SF networks, and the tolerance can be obviously promoted by adjusting b. The benefit from the non-uniform TP strategy deceases with the increase of construction cost, which is proportional to α. The variations of degree distribution and correlation between load and degree are investigated to find the mechanism of these effects. It is found that for smaller b, the cascade of the network is mainly induced by the failures of most high-degree vertices, while for larger b, the cascade attributes to the damage of most low-degree vertices. In addition, the non-uniform TP strategy influences not only the tolerance of the network, but also the degree distribution and the load-degree correlation. Our findings might be hopeful to both network capacity designs and developments of network routing strategies to improve network tolerance. The impacts of the non-uniform TP strategy in other topological networks are in progress [23].

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